AP Calculus Test A
Unit 3 (Differentiation: Composite, Implicit, & Inverse Functions)

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Show all work for credit. Express all answers exactly. Include units when necessary.
Calculators are NOT ALLOWED on this test!

Standard	Problems Assessed	Grade
FUN-3 Recognizing opportunities to apply derivative rules can simplify differentiation.	1-13	1.75

For questions 1-4. Use the table at right.

$\frac{1}{f'(f^{-1}(x))}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	−4	3	6
2	9	4	−5	2
3	5	−2	7	−1

1. $H(x) = g(x) + \sqrt{f(x)}$; $H'(2) =$

$H(x) = g(x) + \sqrt{f(x)}$

$= g(x) + f(x)^{\frac{1}{2}}$

$\cancel{g(2) + 9}$

$H'(x) = g'(x) + \frac{1}{2}f'(x)^{-\frac{1}{2}}$

$H'(2) = g'(2) + \frac{1}{2}f'(2)^{-\frac{1}{2}} = 2 + \frac{1}{2}(4)^{\frac{1}{2}} = 2 + \frac{1}{2}\left(\frac{1}{2}\right) = 2 + 0.25 = 2.25$

2. $J(x) = f(g(x))$; $J'(1) =$

$\cancel{f(g)}$

$f'(3)\,g'(1) = -2 \cdot 6$

$= -12$

3. $K(x) = g(2x - 3)$; $K'(2) =$

$(g'(2(2)-3))\frac{d}{dx}[2x-3]$

$g'(1)(2-0)$

$3 \cdot 2 = 6$

4. $M(x) = \frac{f(x)}{g(x)}$; $M'(3) =$

$(lo)(hi') - (hi)(lo')$

$= \frac{(g(3))(f'(3)) - (f(3))(g'(3))}{g(3)^2}$

$= \frac{((7)(-2)) - (5 \cdot -1)}{7^2} = \frac{-14+5}{49}$

$= -\frac{9}{49}$

Multiple Choice

0.447

5. Differentiate: $y = 3^{(x^2-1)}$

A. $(\ln 3)\, 3^{(x^2-1)}$

B. $(2x)\, 3^{(x^2-1)}$

C. $2x\,(\ln 3)\,(3^{x^2-1})$

D. $x^2(\ln 3)(3^{x^2-1})$

6. Find $\frac{dy}{dx}$ if $y = e^{\sin\sqrt{x}}$

$\cancel{e^{(\sin\sqrt{x})} \cdot (\cos\sqrt{x})\left(\frac{1}{2}x^{-\frac{1}{2}}\right) =}$

A. $(\sin\sqrt{x})e^{\sin\sqrt{x}-1}$

B. $\frac{\cos\sqrt{x}}{2\sqrt{x}}\left(e^{\sin\sqrt{x}}\right)$ e

C. $\frac{e^{\cos\sqrt{x}}}{2\sqrt{x}}$

D. $(\cos\sqrt{x})e^{\sin\sqrt{x}}$

$\frac{1}{\sin\sqrt{x}}$ $\frac{1}{\cos(\sqrt{x})\left(\frac{1}{2}x^{-\frac{1}{2}}\right)} = \frac{1}{\frac{\cos\sqrt{x}}{2}x^{-\frac{1}{2}}}$ $\frac{\cos\sqrt{x}}{2}$ $x^{-\frac{1}{2}}$ $= \frac{\cos\sqrt{x}}{2} \cdot \sqrt{x}$ $\frac{1}{\sqrt{x}}$ $\cos\sqrt{x}$

7. Find $f'(x)$ for $f(x) = \sqrt{5+e^{2x}}$

A. $\frac{xe^{2x-1}}{\sqrt{5+e^{2x}}}$

B. $\frac{1}{2\sqrt{2e^{2x}}}$

C. $\frac{5+e^{2x}}{\sqrt{5+e^{2x}}}$

D. $\frac{e^{2x}}{\sqrt{5+e^{2x}}}$

8. If $y = \tan^{-1}(\cos x)$, then $\frac{dy}{dx} =$

A. $\frac{-\sin x}{1+\cos^2 x}$

B. $-(\sec^{-1}(\cos x))^2 \sin x$

C. $(\sec^{-1}(\cos x))^2$

D. $\frac{1}{(\cos^{-1}x)^2+1}$

9. Let $f(x) = x^4 + 3x - 2$, and let $g(x)$ denote the inverse of f. Then $g'(2)$ is equal to:

A. 7

B. $\frac{1}{7}$ ✓

C. 35

D. $\frac{1}{35}$

$f'(x) = x^4 + 3x - 2$

$= 4x^3 + 3 - 0$

$= 4x^3 + 3$

$2 = x^2 + 3x - 2$

$4 = x^2 + 3x$

$= 1^2 + 3(1) = 1 + 3 = 4$

$g'(2) = \frac{d}{dx}[f^{-1}(2)] = \frac{1}{f'(f^{-1}(2))}$

$\frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{4(1)^4 + 3} = \frac{1}{4+3} = \frac{1}{7}$

10. If $5x^2 + 2x^2y + y^2 = 8$, then $\frac{dy}{dx}$ at the point $(1,1)$ is

A. $-\frac{7}{2}$ ✓

B. $-\frac{2}{7}$

C. $\frac{7}{2}$

D. $\frac{2}{7}$

11. Find the derivative. Simplify your answers when possible. Do not multiply products of binomials and trinomials. Remember to simplify or rewrite before differentiating, if possible. ~~to~~ $ab' + a'b$

a. $f(x) = \sqrt{x^2 - 4x}$

$\frac{1}{2}(x^2 - 4x)^{-\frac{1}{2}}(2x - 4)$

$\neq \frac{1}{2}(x^{-1} - 4x^{-\frac{1}{2}})(2x-4)$

$(\frac{1}{2}x^{-1} - 2x^{-\frac{1}{2}})(2x-4)$

some issues w/ rules of exponents here...

b. $y = (x-1)(x^2+2)^3$

$(\cancel{3(x^2+2)^2)(x^3}$)

$3(x^2+2)^2(2x)$

~~6x~~

$(x-1)(6x(x^2+2)^2) + 1(x^6 + 8)$

$(6x^2 - 6x)(x^2+2)^2 + x^6 + 8$

c. $f(x) = 5^{\sin x}$

~~$5^{\sin x} \ln(5 \sin x)$~~

$5^{\sin x} \ln(5)$ (?)

close!

d. $y = \ln(4x^3 - x^2 + 3)$

$\frac{12x^2 - 2x}{4x^3 - x^2 + 3}$ ✓

$\frac{12x^2 - 2x}{4x^3 - x^2 + 3}$

$(a \cdot b') + (b \cdot a')$

12. If $2x^4 - xy + 3y^3 = 12$, find $\frac{dy}{dx}$.

$$\frac{d}{dx}[xy] = x\frac{dy}{dx} + y$$

$$2x^4 - xy + 3y^3 = 12$$

$$\frac{d}{dx}\left[(2x^4 - xy + 3y^3) = 12\right] : 8x^3 - x\frac{dy}{dx} - y + 9y^2\frac{dy}{dx}$$

$$0 = 8x^3 - x\frac{dy}{dx} - y + 9y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 9y\frac{dy}{dx} = 8x^3 - y$$

$$\frac{dy}{dx}(x - 9y) = 8x^3 - y$$

$$\frac{dy}{dx} = \frac{8x^3 - y}{x - 9y^2}$$ close!

13. Find $\frac{d^2y}{dx^2}$ in terms of x and y, given $x^3 - y^3 = 9$.

$$(lo)(hi') - (hi)(lo')$$

$$x^3 - y^3 = 9$$

$$\frac{d^2y}{dx^2}\left[\frac{x^3}{y}\right] = \qquad \frac{d^2y}{dx^2} = \frac{dy}{dx}\left[\frac{x^3}{y}\right]$$

$$3x^2 - 3y\frac{dy}{dx} = 0$$

$$3x^3 = 3y^2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{(y)(3x^2) - (x^3)\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{3x^3}{3y} = \frac{dy}{dx}$$

$$\frac{x^3}{y} = \frac{dy}{dx}$$

$$= 3x^2y - x^3\left(\frac{x^3}{y}\right)$$

$$= 3x^2y - x^3\frac{x^3}{y}$$

#6

$e^{x} \sin\sqrt{x} \Big] = e^{x}$ 6

$e^{\sin x} \cdot e^{x}$

#7

$f(x) = \sqrt{5+e^{2x}}$

$= (5+e^{2x})^{\frac{1}{2}}$

6561 164025

26 ~~73125~~

$f'(x) = \frac{1}{2}(5+e^{2x})^{-\frac{1}{2}}$

$\frac{1}{2} \cdot \frac{1}{\sqrt{5+e^{2x}}} \cdot e^{2x} = \frac{1}{2\sqrt{2e^{2x}}}$

$\frac{e^{2x}}{\sqrt{5+e^{2x}}} = \frac{2}{2\sqrt{2e^{2x}}}$

$\frac{e^{2x}}{\sqrt{5+e^{2x}}} = \frac{1}{\sqrt{2e^{2x}}}$

$e^{2x} =$

$= \frac{1}{2}(5+e^{2x})^{\frac{1}{2}} \cdot 0 + (\ln e)\, e^{2x}$

54.598

$\frac{\sqrt{5+e^{2x}}}{\sqrt{2e^{2x}}}$

$= \frac{1}{2}(5+e^{2x})^{\frac{1}{2}} \cdot e^{2x} = \frac{1}{2} \cdot \frac{1}{\sqrt{5+e^{2x}}} \cdot e^{2x} \approx 3.536157029$

$\frac{1}{2\sqrt{5+e^{2x}}} \cdot e^{2x}$

$= \frac{e^{2x}}{2\sqrt{5+e^{2x}}}$

$= \frac{e^{2x}}{2\sqrt{5}+2e^{x}}$

$= \frac{1}{2}\sqrt{5} + \frac{1}{2}e^{x} \cdot e^{2x}$

Question #10

$(10)(hi') + (hi)(10')$

$(2x^2)(y) = (y)(4x) + (2x^2)\left(\frac{dy}{dx}\right)$

$5x^2 + 2x^2y + y^2 = 8$

$10x + 4xy + 2x^2\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$2x^2\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = -10x - 4xy$

$\frac{dy}{dx}(2x^2 + 2y) = -10x - 4xy$

$(1,1)$

$\frac{dy}{dx} = \frac{-10x - 4xy}{2x^2 + 2y} = \frac{-10(1) - 4(1)(1)}{2(1)^2 + 2(1)}$

$= \frac{-10-4}{2+2}$

$= \frac{-14}{4}$

$= -3.5$

Question #6

$= \cancel{e^{\sin\sqrt{x}} \cdot (\cos\sqrt{x})\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}$

$= \cancel{e^{\sin\sqrt{x}} \cdot \left(\frac{\cos\sqrt{x}}{2} \cdot \frac{1}{\sqrt{x}}\right)}$

$= \cancel{e^{\sin\sqrt{x}}} \cdot \frac{\cos\sqrt{x}}{2\sqrt{x}}$